Sample Question Paper - 16
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

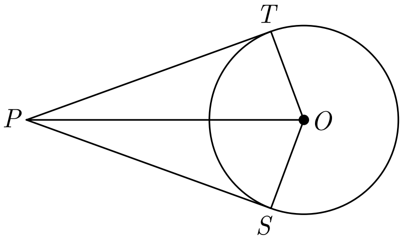
1. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

OR

Find the nature of roots of the quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$.

2. If the n^{th} term of a sequence is $3 - 2n$. Find the sum of fifteen terms.

3. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



4. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm^3. Use $\pi = \frac{22}{7}$

5. Find the unknown entries a, b, c, d in the following distribution of heights of students in a class :

Height (in cm)	Frequency	Cumulative Frequency
150-155	12	12
155-160	a	25
160-165	10	b
165-170	c	43
170-175	5	48
175-180	2	d

6. Find the mode of the following distribution :

Classes	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

OR

Consider the following distribution :

Marks Obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
Number of students	63	58	55	51	48	42

(i) Calculate the frequency of the class 30 - 40.
(ii) Calculate the class mark of the class 10 - 25.

Section B

7. Solve for x :

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}; x \neq 1,-2,2$$

8. If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

9. A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of the top of a hill as 30° and the angle of depression of the base of the hill as 60°. Calculate the distance of the hill from the platform and the height of the hill.

10. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

OR

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Section C

11. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively. Find the height of multi-storey building and distance between two buildings.

12. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

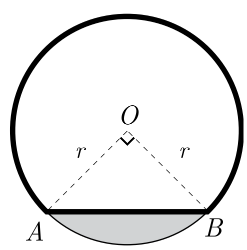
OR

Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

13. Atal Tunnel (also known as Rohtang Tunnel) is a highway tunnel built under the Rohtang Pass in the eastern Pir Panjal range of the Himalayas on the Leh-Manali Highway in Himachal Pradesh. At a length of 9.02 km, it is the longest tunnel above 10,000 feet (3,048 m) in the world and is named after former Prime Minister of India, Atal Bihari Vajpayee. The tunnel reduces the travel time and overall distance between Manali and Keylong on

the way to Leh. Moreover, the tunnel bypasses most of the sites that were prone to road blockades, avalanches, and traffic snarls.

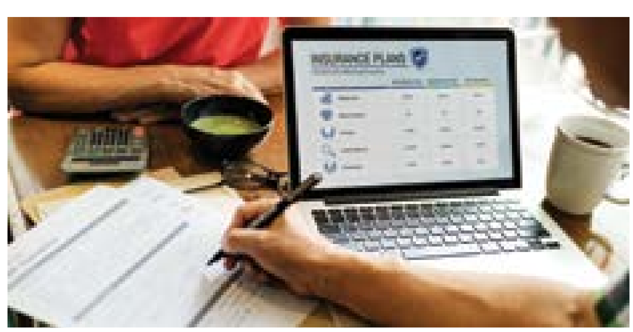
Earth is excavated to make a railway tunnel. The tunnel is a cylinder of radius 7 m and length 450 m. A level surface is laid inside the tunnel to carry the railway lines. Figure given below shows the circular cross - section of the tunnel. The level surface is represented by AB, the centre of the circle is O and $\angle AOB = 90^\circ$. The space below AB is filled with rubble (debris from the demolition buildings).



(i) How much volume of earth is removed to make the tunnel ?

(ii) A coating is to be done on the surface of inner curved part of tunnel. What is the area of tunnel to be being coated ?

14. Life insurance is a contract between an insurance policy holder and an insurer or assurer, where the insurer promises to pay a designated beneficiary a sum of money upon the death of an insured person (often the policy holder). Depending on the contract, other events such as terminal illness or critical illness can also trigger payment. The policy holder typically pays a premium, either regularly or as one lump sum.



SBI life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

(i) What is the median value of age ?
(ii) What is the mode value of age ?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

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SECTION A

1. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

Ans :

We have $\quad ax^2 + 7x + b = 0 \quad (1)$

Substituting $x = \frac{2}{3}$ in above equation we obtain

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

$$4a + 42 + 9b = 0$$

$$4a + 9b = -42 \quad (2)$$

and substituting $x = -3$ in (1) we obtain

$$9a - 21 + b = 0$$

$$9a + b = 21 \quad (3)$$

Solving (2) and (3), we get $a = 3$ and $b = -6$

OR

Find the nature of roots of the quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$.

Ans :

We have $\quad 2x^2 - \sqrt{5x} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,

Now $\quad b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1)$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots i.e., imaginary roots.

2. If the n^{th} term of a sequence is $3 - 2n$. Find the sum of fifteen terms.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n

Here, $a_n = 3 - 2n$

Taking $n = 1$, $\quad a_1 = 3 - 2 = 1$

15th term, $\quad a_{15} = 3 - 2 \times 15 = 3 - 30 = -27$

Now $\quad S_n = \frac{n}{2}(a_1 + a_n)$

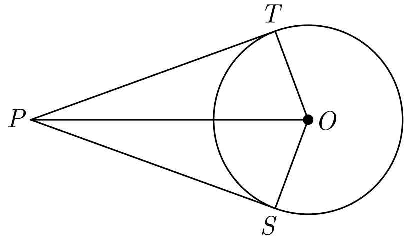
$$S_{15} = \frac{15}{2}(a_1 + a_{15})$$

$$= \frac{15}{2}[1 + (-27)]$$

$$= \frac{15}{2}[-26]$$

$$= 15 \times (-13) = -195$$

3. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



Ans :

We have $\quad \angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

$$\frac{1}{2} = \frac{PS}{OP}$$

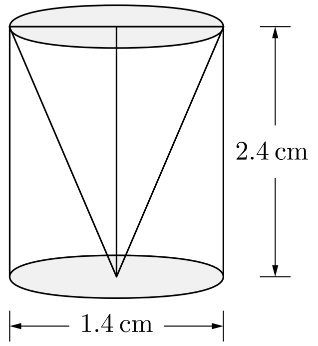
$$OP = 2PS$$

Hence proved.

4. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm^3. Use $\pi = \frac{22}{7}$

Ans :

As per question the figure is shown below.



Volume of remaining solid is difference of volume of cylinder and volume of cone.

$$\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times (0.7)^2 \times 2.4$$

$$= 44 \times 0.1 \times 0.7 \times 0.8$$

$$= 4.4 \times .56 = 2.464 \text{ cm}^3.$$

5. Find the unknown entries a, b, c, d in the following distribution of heights of students in a class :

Height (in cm)	Frequency	Cumulative Frequency
150-155	12	12
155-160	a	25
160-165	10	b
165-170	c	43
170-175	5	48
175-180	2	d

Ans :

From the table,

$$12 + a = 25 \Rightarrow a = 25 - 12 = 13$$

$$25 + 10 = b \Rightarrow b = 35,$$

$$b + c = 43 \Rightarrow$$

$$c = 43 - b = 13 - 35 = 8$$

and $48 + 2 = d \Rightarrow d = 50$

6. Find the mode of the following distribution :

Classes	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Ans :

Class 35-40 has the maximum frequency 50, therefore this is model class.

Now $l = 35$, $f_1 = 50$, $f_2 = 42$, $f_0 = 34$, $h = 5$

$$\text{Mode, } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 35 + \frac{50 - 34}{100 - 34 - 42} \times 5$$

$$= 35 + \frac{16 \times 5}{24} = 38.33$$

OR

Consider the following distribution :

Marks Obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
Number of students	63	58	55	51	48	42

(i) Calculate the frequency of the class 30 - 40.
(ii) Calculate the class mark of the class 10 - 25.

Ans :

Class Interval	c.f.	f
0-10	63	5
10-20	58	3
20-30	55	4
30-40	51	3
40-50	48	6
50-60	42	42

(i) Frequency of the class 30 - 40 is 3.

(ii) Class mark of the class :$10 - 25 = \frac{10 + 25}{2}$

$$= \frac{35}{2} = 17.5$$

Section B

7. Solve for x :

$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$

Ans :

We have $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$

$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$5x^2+19x-30 = 0$$

$$(5x-6)(x+5) = 0$$

$$x = -5, \frac{6}{5}$$

8. If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ (1)

$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7}$ (2)

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$

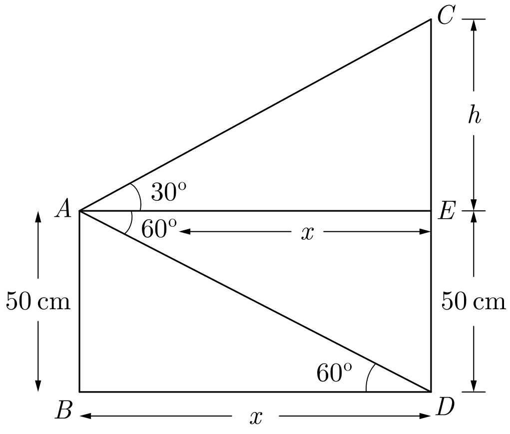
$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$.

9. A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of the top of a hill as 30° and the angle of depression of the base of the hill as 60°. Calculate the distance of the hill from the platform and the height of the hill.

Ans :

Let AB be the wooden platform of height 50 m. As per question we have shown the figure below. Here total height of hill is CD and h is the height of hill above platform.



Now, $CD = CE + ED$

$$= (h + 50) \text{ m}$$

$$BD = AE = x$$

In ΔABD, $\tan 60° = \frac{AB}{BD}$

$$\sqrt{3} = \frac{50}{x}$$

$$x = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3} \text{ m}$$

In ΔCEA, $\tan 30° = \frac{CE}{AE}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} = \frac{50\sqrt{3}}{3} \times \frac{1}{\sqrt{3}} = \frac{50}{3} \text{ m}$$

Now, $CD = h + 50$

$$= \frac{50}{3} + 50$$

$$= \frac{200}{3} \text{ m} = 66.66 \text{ m}$$

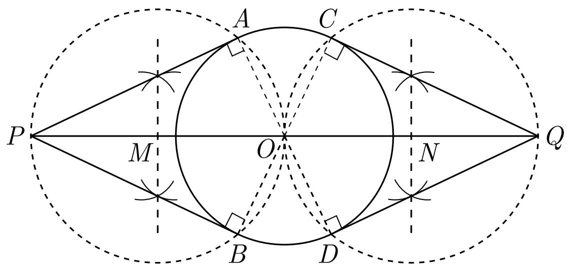
So, distance between hill and platform is $\frac{50\sqrt{3}}{3}$ m and height of hill is 66.66 m.

10. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

Ans :

Steps of Construction :

1. Draw a circle of radius 4 cm with centre O and draw a diameter.
2. Extend its diameter on both sides and cut $OP = OQ = 9$ cm.
3. Bisect PO such that M be its mid-point.
4. Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at A and B.
5. Join PA and PB.
   Thus, PA and PB are the two required tangents from P.
6. Now bisect OQ such that N is its mid-point.
7. Taking N as centre and NO as radius, draw a circle. Let it intersect the given circle at C and D.
8. Join QC and QD.
   Thus, QC and QD are the required tangents from Q.



Justification :

Join OA to get,

$$\angle OAP = 90^\circ$$

(Angle in a semi-circle)

Since $PA \perp OA$, thus PA is a tangent.

Similarly, $PB \perp OA$ PB is a tangent.

Now, join OC to get,

$$\angle QCO = 90^\circ \quad \text{(Angle in a semi-circle)}$$

Since $QC \perp OC$, thus QC is a tangent.

Similarly, $QD \perp OC$, thus QD is a tangent.

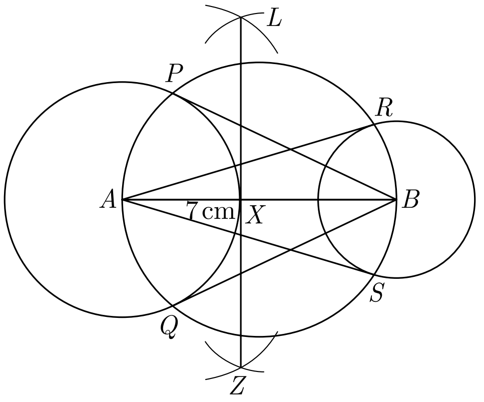
OR

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Ans :

Steps of construction :

1. Draw a line segment AB of length 7 cm.
2. Draw a circle with A as centre and radius 3 cm.
3. Draw another circle with B as centre and radius 2 cm.



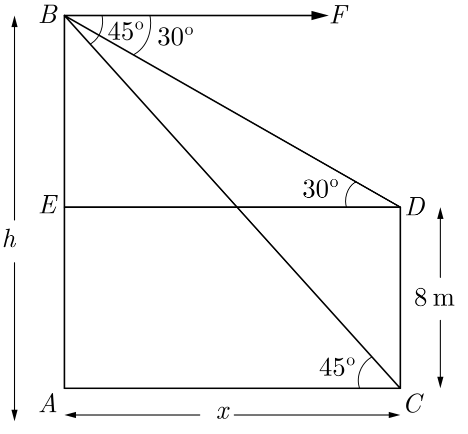
4. Draw another circle taking AB as diameter circle, which intersects first two circles at P and Q, R and S.
5. Join B to P, B to Q A to R and A to S.
   Hence, BP, BQ, AR and AS are the required tangents.

Section C

11. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively. Find the height of multi-storey building and distance between two buildings.

Ans :

As per given in question we have drawn figure below.



Here $AE = CD = 8$ m

$$BE = AB - AE = (h - 8)$$

and $\quad AC = DE = x$

Also, $\quad \angle FBD = \angle BDE = 30^\circ$

$$\angle FBC = \angle BCA = 45^\circ$$

In right angled $\Delta\, CAB$ we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$1 = \frac{h}{x} \Rightarrow x = h \qquad ...(1)$$

In right angled $\Delta\, EDB$

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{h-8}{x}$$

$$x = \sqrt{3}(h-8) \qquad ...(2)$$

From (1) and (2), we get

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$8\sqrt{3} = \sqrt{3}h - h$$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 4\sqrt{3}(\sqrt{3}+1) = (12+4\sqrt{3}) \text{ m}$$

Since, $x = h$, $\quad x = (12+4\sqrt{3})$

$$\text{Distance} = (12+4\sqrt{3}) \text{ m}$$

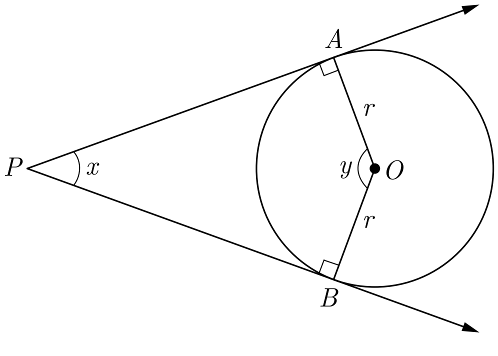
$$= 4(3+\sqrt{3}) \text{ m}$$

Hence the height of multi storey building is $4(3+\sqrt{3})$ m.

12. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

Ans :

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\quad \angle A = \angle B = 90^\circ$

Since, $AOBP$ is a quadrilateral,

$$\angle A + \angle B + x + y = 360^\circ$$

$$90^\circ + 90^\circ + x + y = 360^\circ$$

$$180 + x + y = 360^\circ$$

$$x + y = 180^\circ$$

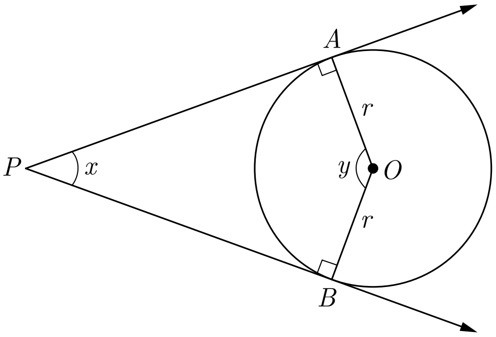
Therefore opposite angle are supplementary.

OR

Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

Ans :

As per question we draw figure shown below.



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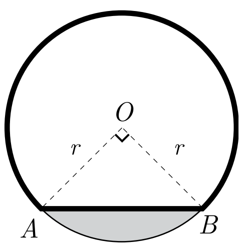
Earth is excavated to make a railway tunnel. The tunnel is a cylinder of radius 7 m and length 450 m. A level surface is laid inside the tunnel to carry the railway lines. Figure given below shows the circular cross - section of the tunnel. The level surface is represented by AB, the centre of the circle is O and $\angle AOB = 90°$. The space below AB is filled with rubble (debris from the demolition buildings).



(i) How much volume of earth is removed to make the tunnel ?

(ii) A coating is to be done on the surface of inner curved part of tunnel. What is the area of tunnel to be being coated ?

Ans :

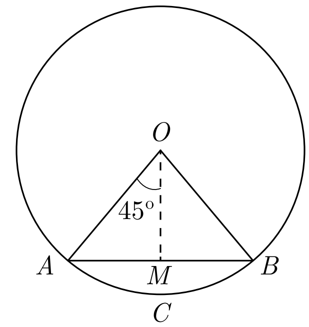
(i) Cross-section area of tunnel to be excavated $= \pi r^2$

Volume of earth to be removed,

$$\pi r^2 l = \frac{22}{7} \times 7 \times 7 \times 450$$

$$= 69300 \text{ m}^3$$

(ii) The geometry of cross-section is shown below.



Triangle OAB is isosceles triangle having right angle at O.

Length of curved part of cross-section,

$$= \frac{2\pi r(360° - 90°)}{360°}$$

$$= \frac{2 \times \frac{22}{7} \times 7(360° - 90°)}{360°}$$

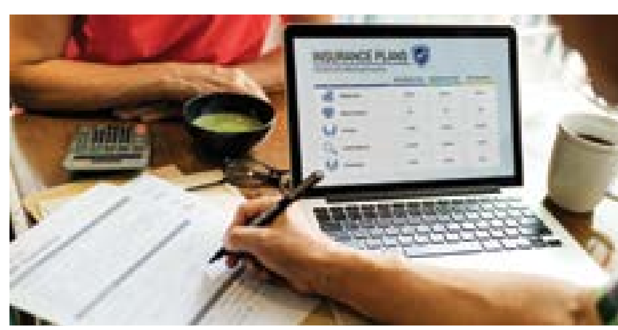
$$= \frac{2 \times 22 \times 270°}{360°} = 33 \text{ m}$$

Total curved surface area of tunnel

= Length of curved part of cross-section × Length of tunnel

$$= 33 \times 450 = 14850 \text{ m}^2$$

14. Life insurance is a contract between an insurance policy holder and an insurer or assurer, where the insurer promises to pay a designated beneficiary a sum of money upon the death of an insured person (often the policy holder). Depending on the contract, other events such as terminal illness or critical illness can also trigger payment. The policy holder typically pays a premium, either regularly or as one lump sum.



SBI life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2

Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

(i) What is the median value of age ?
(ii) What is the mode value of age ?

Ans :

The given table is cumulative frequency distribution. We write the frequency distribution as given below:

Class interval	Cumulative Frequency	Frequency
15-20	2	$2-0=2$
20-25	6	$6-2=4$
25-30	24	$24-6=18$
30-35	45	$45-24=21$
35-40	78	$78-45=33$
40-45	89	$89-78=11$
45-50	92	$92-89=3$
50-55	98	$98-92=6$
55-60	100	$100-98=2$

We have, $\sum f_i = N = 100$

(i) Cumulative frequency just greater than $\frac{N}{2} = \frac{100}{2} = 50$ is 78 and the corresponding class is 35-40. Thus median class is 35-40.
Now, $l = 35$, $\frac{N}{2} = 50$, $F = 45$, $f = 33$ and $h = 5$

Median,
$$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$
$$= 35 + \left[\frac{50-45}{33}\right] \times 5$$
$$= 35 + \frac{25}{33}$$
$$= 35 + 0.76 = 35.76 \text{ years}$$

Thus, the median age 35.76 years.

(ii) Now $l = 35$, $f_1 = 33$, $f_2 = 11$, $f_0 = 21$, $h = 5$

Mode,
$$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$
$$= 35 + \frac{33-21}{66-21-11} \times 5$$
$$= 35 + \frac{12}{33} \times 5$$
$$= 35 + \frac{20}{11}$$
$$= 35 + 1.82 = 36.82 \text{ years}$$